Violation of Bell's inequality with entangled photon pairs

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Abstract

Photon pairs deflected by polarizing lenses provide an opportunity to show the violation of Bell's inequality by experimentally proving the existence of quantum entanglement. By setting the lenses to predetermined angles and measuring the deflected photons' polarization, the correlator $E(\alpha, \beta)$ of each measurement can be calculated. Using this, the S-parameter S can be deduced, which can not exceed the value of 2, granted a hidden variable theorem exists. Following this experiment, the results of $S_{t=1} = 2.62 \pm 0.40$ and $S_{t=2.5} = 2.58 \pm 0.25$ was achieved for a laser time of 1 s and 2.5 s respectively. Taking uncertainties into account, both values exceed the posed limit by the inequality. This shows a clear violation of Bell's inequality and therefore displays empirical evidence of quantum entanglement and disproves the presence of a hidden variable theorem.

1 Introduction

Einstein was a strong believer in determinism and refused the thought of (sub)atomic particles acting probabilistically. This is shown in Einstein's 'hidden variables theorem', in which he claims that instead of acting probabilistically, particles inhibit deterministic properties that we have not found yet, or have remained 'hidden' that can describe and predict the ways particles will behave. A prime example of probabilistic behavior is quantum entanglement. Loosely speaking, this is the property of two particles being inherently 'connected' with one another, despite the distance between them being arbitrarily large. More formally speaking, it disproves the principle of information being locally contained by particles being in a correlated state independent of distance. Einstein believed entanglement to be explainable by hidden variables coming in to play and therefore follows a certain set of statistical laws. This experiment will repeat a monumental experiment performed in 1972 according to theory developed by John Stewart Bell in 1964, showing that entanglement is completely independent of hidden variables and validating that information is indeed not locally contained.

2 Theory

2.1 Entangled photon pairs

Two particles are entangled when the state of one particle is directly correlated to the state of the other particle. Quantum states are not absolute; particles only find themselves in a given state once measured, meaning measuring one entangled state will immediately cause the other state to become known. The appurtenant states for this experiment are the polarization states of photon pairs, which are labeled H and V for horizontal and vertical polarization respectively.

2.2 CHSH Inequality

Bell's inequality takes the form of the CHSH inequality in this experiment:

$$|S| \leq 2 \tag{1}$$

With S being the S-parameter, defined by:

$$S := E(\alpha_1, \beta_1) - E(\alpha_1, \beta_2) + E(\alpha_2, \beta_1) + E(\alpha_2, \beta_2) \tag{2}$$

Here, the values $E(\alpha_0, \beta_0)$ are called quantum correlators of particle pairs, defined as the statistical average of the product of the expected outcomes. The core idea behind this experiment is that the S-parameter's maximum limitation of 2 given by equation 1 is a way to quantify the existence of a local hidden-variable theory (J.F. Clauser; M.A. Horne; A. Shimony; R.A. Holt, 1969). If one such theory exists, statistical rules imply that this S-parameter is bounded by -2 and 2 as shown in the CHSH inequality. If no such theory exists however, it is predicted that the maximum value S can become is $2\sqrt{2}$, commonly referred to in literature as Tsirelson's bound (Cirel'son, B. S., March 1980).

Figure 1 below explains how a Bell's experiment works. First, a source emits two entangled photons, aptly labeled 1 and 2. Both photons are sent to their respective polarizers labeled PBS1 and PBS2 respectively. The axes of the polarizers are labeled the vectors $\vec{a}$ for PBS1 and $\vec{b}$ for PBS2 with θ_1 and θ_2 being the angle between the vectors and their vertical axis. The polarizer consists of a beam splitter, sending the particle to single-photon detectors D(0) and D(1).

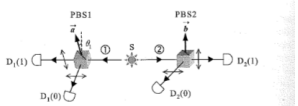


Figure 1: Setup for a Bell's experiment.

Now we define the probability of a detection of a photon by these single-photon detectors. We define the probabilities using quantum mechanics:

$$P_{11}(\theta_1, \theta_2) = \frac{1}{2}\cos^2(\theta_1 - \theta_2) \tag{3}$$

$$P_{10}(\theta_1, \theta_2) = \frac{1}{2}\sin^2(\theta_1 - \theta_2) \tag{4}$$

$$P_{01}(\theta_1, \theta_2) = \frac{1}{2}\sin^2(\theta_1 - \theta_2) \tag{5}$$

$$P_{00}(\theta_1, \theta_2) = \frac{1}{2}\cos^2(\theta_1 - \theta_2) \tag{6}$$

Where P_{11} is the probability that $D_1(1)$ and $D_2(1)$ detect a photon, P_{10} is the probability that $D_1(1)$ and $D_2(0)$ detect a photon, P_{01} is the probability that $D_1(0)$ and $D_2(1)$ detect a photon and P_{00} is the probability that $D_1(0)$ and $D_2(0)$ detect a photon. With these probabilities we can find a parameter S given in equation 1 to define Bell's inequality.

$$S := E(\theta_1, \theta_2) - E(\theta_1, \theta_2') + E(\theta_1', \theta_2) + E(\theta_1', \theta_2') \tag{7}$$

where

$$E(\theta_1, \theta_2) = P_{11}(\theta_1, \theta_2) + P_{10}(\theta_1, \theta_2) + P_{01}(\theta_1, \theta_2) + P_{00}(\theta_1, \theta_2) \tag{8}$$

In order to test the validity of the CHSH inequality, we need to set the angles to specific degrees. These angles are as follows: $\theta_1 = 0°$, $\theta_2 = 22.5°$, $\theta_1' = 45°$ and $\theta_2' = 67.5°$.

3 Method

3.1 experimental setup

For the experiment an optical unit called quED was used to generate and measure the entangled photon pairs. Our setup can be seen in the figure below.

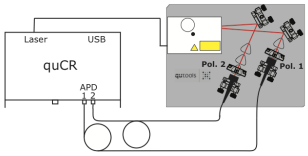


Figure 2: The experimental setup.

To generate entangled photon pairs, a process called spontaneous parametric down-conversion, or SPDC for short, is used in the quED. In this process, a high-intensity laser beam is shot trough a nonlinear crystal which generates low-frequency entangled photon pairs. After creation, these photons split and

individually go trough 2 different polarizers with adjustable angles. Here, the optical fibers will detect the single photons and analyze their polarization. These fibers will send the photon counts on a digital screen where the counts will be shown. (quED – Qutools, z.d.) In contrast to the setup discussed in the theory, the optical fibers can only measure the polarization in one direction, namely the H polarization. To solve this problem, the angles have to be adjusted with 90° in order to be able to measure the V polarization as well.

3.2 Measurement procedure

Each polarizer was set to the predetermined angle and the amount of photons in each state was measured. The predetermined angles for polarizer α and β respectively are $\alpha_1 = 0°$, $\alpha_2 = 45°$, $\beta_1 = 22.5°$ and $\beta_1 = 67.5°$. As many measurements were done with one given angle setting in order to limit the amount of times the lens needed to be adjusted, in order to nullify any small but significant angle uncertainties. The sensors then measured the amount of observed photons in the given states of HH, VV, HV and VH. As stated in the experimental setup, the angles have to be adjusted by 90° to measure in the vertical (V) direction.

HH polarization:	No angles need to be adjusted.
HV polarization:	β angle must be adjusted by 90°.
VH polarization:	α angle must be adjusted by 90°.
VV polarization:	α and β angles must be adjusted by 90°.

This experiment was repeated twice; once for the laser being on for t = 1.0 s, the other for t = 2.5 s. As specified in section 2.2, the quantum correlators $E(\alpha, \beta)$ are defined as the statistical average of the product of the expected outcomes. In this case, this definition of the quantum correlators is condensed down to the following formula, which is used to calculate the respective correlator values from empirical data.

$$E(\alpha, \beta) = \frac{N_{VV}(\alpha, \beta) + N_{HH}(\alpha, \beta) - N_{VH}(\alpha, \beta) - N_{HV}(\alpha, \beta)}{N_{tot}} \tag{9}$$

Where $N_{XY}(\alpha, \beta)$ is the number of counts measured for the respective polarization positions X and Y at the sensor angles α and β and N_{tot} being the total number of photon counts for the given angles. The angles were adjusted accordingly in order to get the four correlators $E(\alpha_1, \beta_1)$, $E(\alpha_1, \beta_2)$, $E(\alpha_2, \beta_1)$ and $E(\alpha_2, \beta_2)$. Finally, using equation 2 these correlators were used to calculate the S-parameter.

4 Results

The calculated correlator values E_n can be found in the following table:

Time t (s)	E_1	E_2	E_3	E_4
1.0	0.677	-0.6722	0.644	0.592
2.5	0.628	-0.688	0.684	0.618

As shown in 1, the S-parameter can not exceed S = 2 if a hidden variable theorem holds true. If no such theorem exists, the S-parameter can theoretically go as high as $2\sqrt{2} \approx 2.828$. Our results are shown in the following table:

Time t (s)	S-parameter	Uncertainty	Minimum S-parameter
1.0	2.585	0.402	2.183
2.5	2.618	0.253	2.365

The minimum S-parameter was calculated by subtracting the uncertainty from the S-parameter, meaning this value is the lowest S can be. The calculations for the uncertainties can be found in the appendix.

The best result was found with an time of 2.5 seconds that resulted in a value of $S = 2.62 \pm 0.25$. The time of 1.0 seconds resulted in a value of $S = 2.59 \pm 0.40$.

5 Discussion

Overall the experiment went as expected. The uncertainties achieved were generally large, but fell well into the acceptable margin. The experiment went without any major complications, although some small improvements could be made during the experiment itself. External photons (photons not produced by the laser) could be further limited by eliminating all the light in the room for example, or reproducing the test underground where external photons are more scarce. A temperature controlled environment or utilizing a better vibration-dampening board are also examples to make the measurement more accurate.

6 Conclusion

The goal of the experiment was achieved by using an optics setup while adjusting the angle and counting the amount of photons on different angle combinations. Ultimately, a value of $S = 2.61 \pm 0.25$ and $S = 2.59 \pm 0.40$ were achieved for 2.5 and 1.0 seconds respectively, both of which are above the CHSH inequality limit of 2. As a result it can be concluded that any hidden variable theorems such as the one posed by Einstein are not present and therefore that information on the quantum scale is not localized. By eliminating more external photons, using a more temperature controlled environment and utilizing better a vibration dampening board the value of S could be improved, further solidifying the

violation. Future experiments can be done by using a setup that spans a larger distance between detectors or by measuring other forms of entangled particles, which could ultimately expand our knowledge regarding the quantum realm.

7 Appendix

The number of photon counts follows a Poisson distribution, meaning the uncertainty scales with $\sqrt{x}$. Following this, the uncertainty of the number of photon counts N:

$$u(N) = \sqrt{N} \tag{10}$$

The calculation for the uncertainty of $E(\alpha, \beta)$ was done with the calculus approach:

$$u(E(\alpha, \beta)) = \sqrt{\frac{1 + E(\alpha, \beta)^2}{N}} \tag{11}$$

To get the uncertainty of S we can simply sum up the 4 different values of $E(\alpha, \beta)$:

$$u(S) = u(E(\alpha_1, \beta_1)) + u(E(\alpha_1, \beta_2)) + u(E(\alpha_2, \beta_1)) + u(E(\alpha_2, \beta_2)) \tag{12}$$

8 Bibliography

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